Lomb-Scargle Phase Shift Formula and Examples

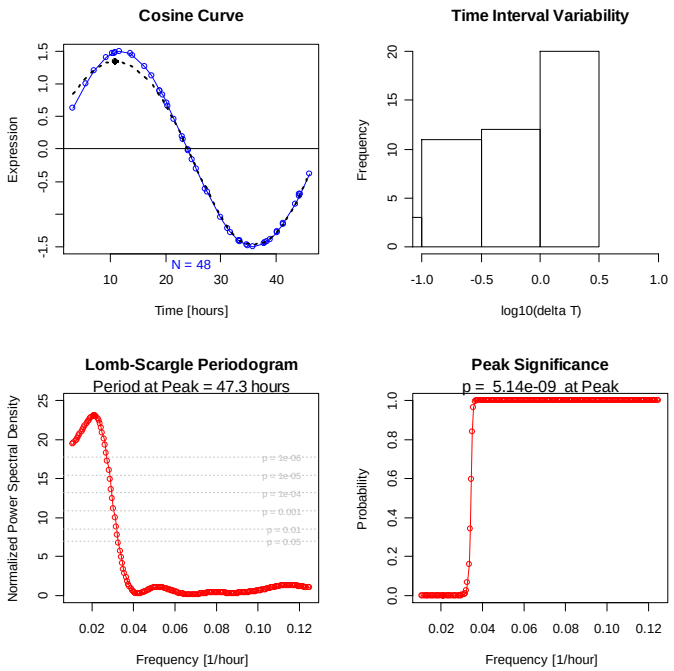
efg, 17 August 2007

This analysis assumes Lomb-Scargle code is available from the LombScargle.zip from http://research.stowers-institute.org/efg/2005/LombScargle/R/index.htm.

```
source("LombScargle.R")
```

Run this R Code:

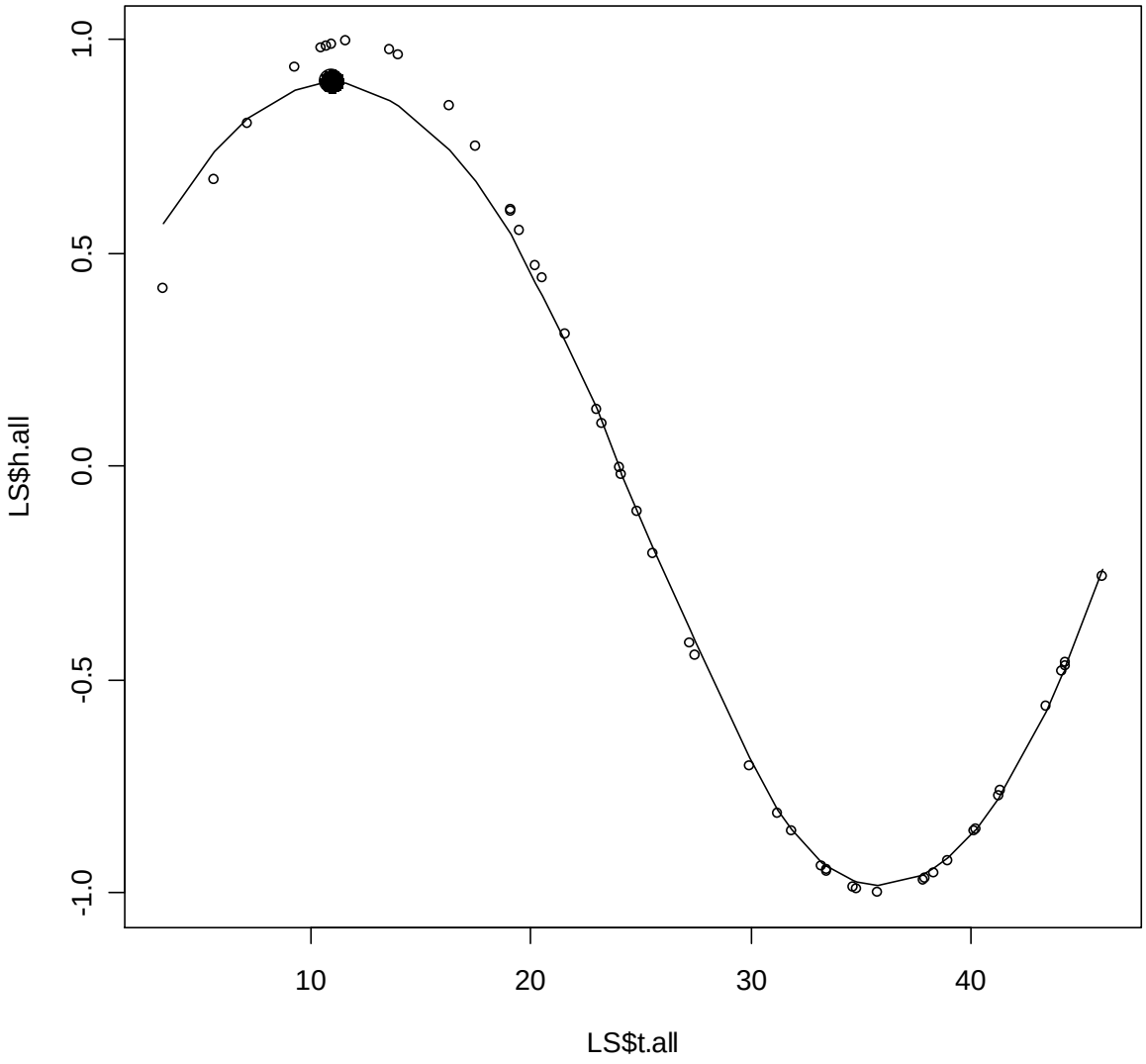
```
unit <<- "hour" # hourly data
set.seed(19)    # make this example reproducible
time <- sort( runif(48, 0, 48) )

Amplitude <- 1.5
PeriodExact <- 48
PhaseShiftExact <- pi/2
Expression <- Amplitude * cos(2*pi*time/PeriodExact  - PhaseShiftExact)

M <- 4*length(time) # often 2 or 4

# Use test frequencies corresponding to
# periods from 8 hours to 96 hours
TestFrequencies <- (1/96) + (1/8 - 1/96) * (1:M / M)

# Use Horne & Baliunas' estimate of independent frequencies
Nindependent <- NHorneBaliunas(length(Expression))

LS <- ComputeAndPlotLombScargle(time, Expression,
                                TestFrequencies, Nindependent,
                                "Cosine Curve")
```



At this point the LS list has several variables of interest, which can be plotted:

```
plot(LS$t.all, LS$h.all)
lines(LS$t.all, LS$h.loess)
points(LS$h.peak$maximum, LS$h.peak$objective, pch=PCH_CIRCLE, col="black", cex=2)
```

Plot the original raw data points (circles), and the loess smoothed curve (black line). The "optimal" point is shown by a black point



This "optimal" point is at (LS$h.peak$maximum, LS$h.peak$objective)

```
> LS$h.peak$maximum    TimeOffset
[1] 10.98842
> LS$h.peak$objective
[1] 0.9016772
```

The Lomb-Scargle period is given by

```
> LS$PeakPeriod
[1] 47.26154
```

The maximum of the original time series is probably the best to determine the amplitude of the cosine curve.

```
> Amplitude <- max(LS$h.all)
> Amplitude
[1] 1.498046
```

A cosine curve based on the Lomb-Scargle analysis is:

$$y = Amplitude * \cos \frac{2 * p * Time - 2 * p * TimeOffset}{Period}$$

$$y = Amplitude * \cos \frac{2 * p * (Time - TimeOffset)}{Period}$$

The phase shift in 0 .. 2p can be defined:

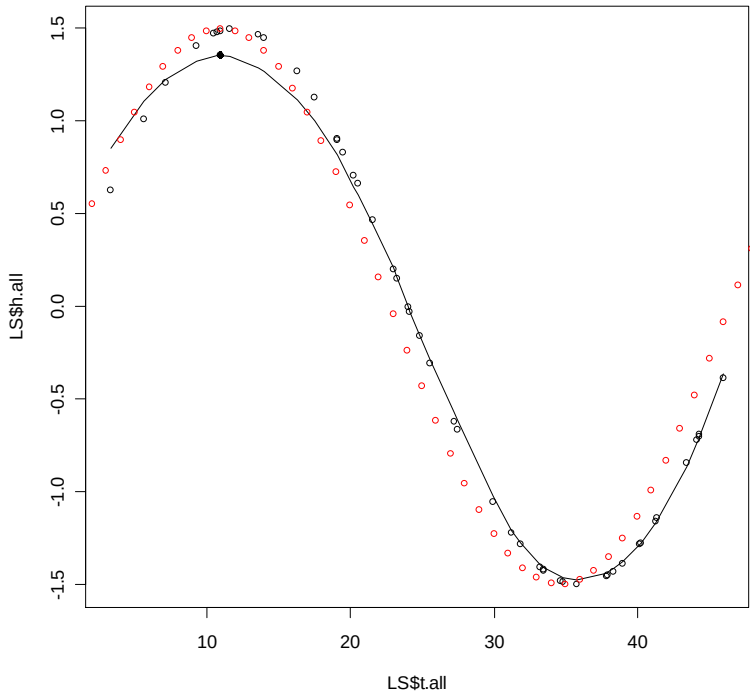
$$PhaseShift = \frac{2 * p * TimeOffset}{Period}$$

An alternative form of the equation is:

$$y = Amplitude * \cos\left(\frac{2 * p * Time}{Period} - PhaseShift \right)$$

This "mathematical curve" can be overlaid on the original plot (in red):

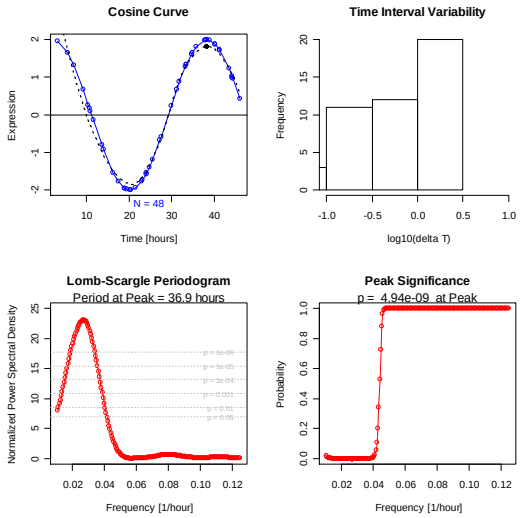
```
> plot(LS$t.all, LS$h.all)
> lines(LS$t.all, LS$h.loess)
> points(LS$h.peak$maximum,LS$h.peak$objective, pch=PCH_CIRCLE, col="black")
>
> Period <- LS$PeakPeriod
> Period
[1] 47.26154
>
> Amplitude <- max(LS$h.all)
> Amplitude
[1] 1.498046
>
> T <- 0:48
> y <- Amplitude * cos(2*pi*(T - LS$h.peak$maximum) / LS$PeakPeriod)
> points(T, y, col="red")
>
> PhaseShift <- 2*pi*LS$h.peak$maximum / LS$PeakPeriod
> PhaseShift
[1] 1.460855
>
> PhaseShiftError <- PhaseShiftExact - PhaseShift
> PhaseShiftErrorPercent <- 100 * PhaseShiftError / PhaseShiftExact
> PhaseShiftError
[1] 0.1099415
> PhaseShiftErrorPercent
[1] 6.999091
```

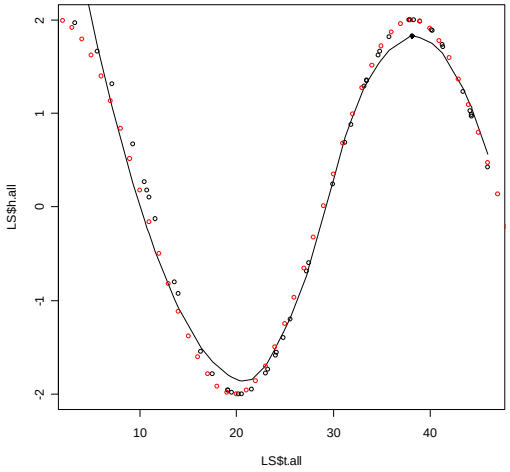


Perhaps, a more exact time shift could be found by shifting the red curve above and finding an optimal correlation with the original data.

Other Examples:

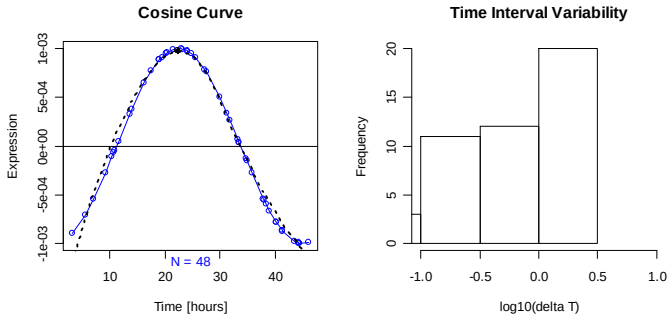
```
Amplitude <- 2.0
PeriodExact <- 36
PhaseShiftExact <- pi/8
Expression <- Amplitude * cos(2*pi*time/PeriodExact  - PhaseShiftExact)
```

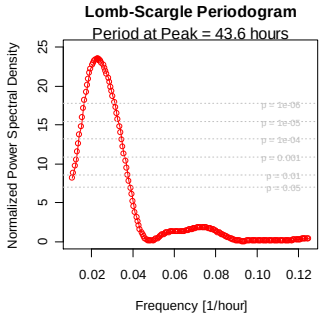


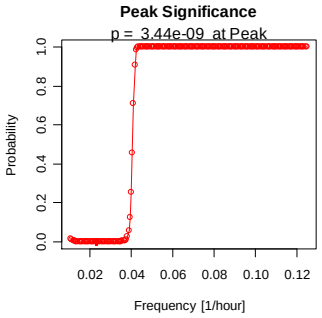


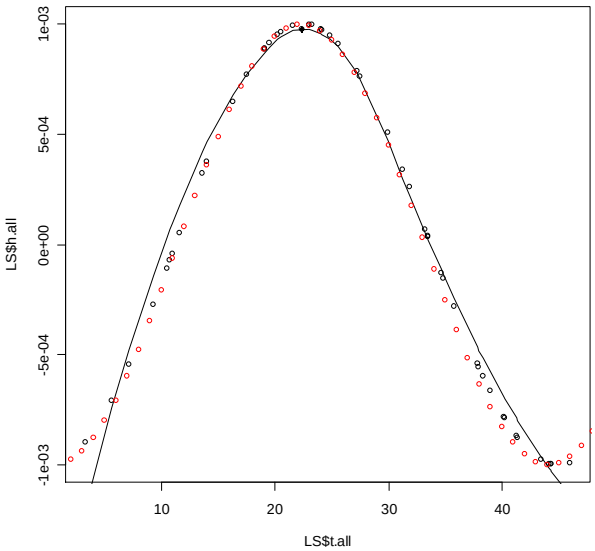

```
> Period
[1] 36.864
> Amplitude
[1] 1.999809
> PhaseShift    # answer can have extra 2p
[1] 6.50696

> pi/8 + 2*pi
[1] 6.675884

% Error (adjusted) = -2.53
```

```
Amplitude <- 0.001
PeriodExact <- 45
PhaseShiftExact <- pi
Expression <- Amplitude * cos(2*pi*time/PeriodExact  - PhaseShiftExact)
```





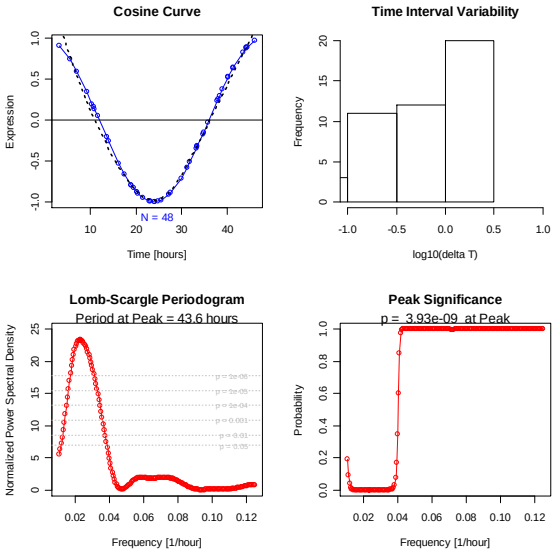


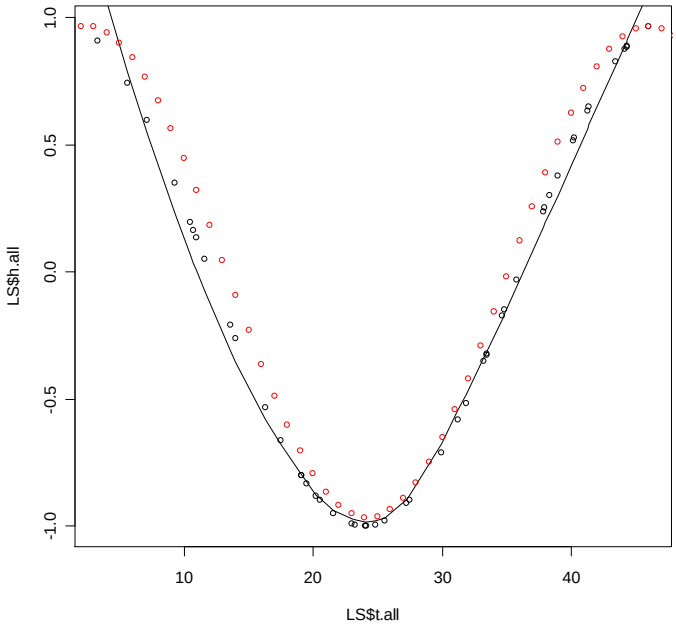



```
> Amplitude
[1] 0.0009977873
> PhaseShift
[1] 3.221355
> PhaseShiftError
[1] -0.07976247
> PhaseShiftErrorPercent
[1] -2.538918
```

Lomb-Scargle Phase Shift Formula and Examples

```
Amplitude <- 1
PeriodExact <- 48
PhaseShiftExact <- 0
Expression <- Amplitude * cos(2*pi*time/PeriodExact  - PhaseShiftExact)
```






```
> Period
[1] 43.57447
> Amplitude
[1] 0.9665584
> PhaseShift  # answer can have extra 2p
[1] 6.63563

> 2*pi
[1] 6.283185
```